SOLUCIONES A LA ECUACIÓN DE LAPLACE

I. En coordenadas rectangulares.

$\Phi(x,y,z)=\sum_{nm} A_{nm}\, X_{nm}(x) Y_{nm}(y) Z_{nm}(z)$, donde $\mp k_x^2(n,m)\mp k_y^2(n,m)\mp k_z^2(n,m)=0$, y:

Autovalores	$X_{nm}(x)$	$Y_{nm}(y)$	$Z_{nm}(z)$
$k^2(n,m)=0$	$X(x)=A+Bx$	$Y(y)=C+Dy$	$Z(z)=E+Fz$
$-k^2(n,m)\neq 0$	$\operatorname{senh}[k_x(n,m)x]$ $\cosh[k_x(n,m)x]$ $\exp[-k_x(n,m)x]$ $\exp[+k_x(n,m)x]$	$\operatorname{senh}[k_y(n,m)y]$ $\cosh[k_y(n,m)y]$ $\exp[-k_y(n,m)y]$ $\exp[+k_y(n,m)y]$	$\operatorname{sen\,h}[k_z(n,m)z]$ $\cosh[k_z(n,m)z]$ $\exp[-k_z(n,m)z]$ $\exp[+k_z(n,m)z]$
$+k^2(n,m)\neq 0$	$\operatorname{sen}[k_x(n,m)x]$ $\cos[k_x(n,m)x]$	$\operatorname{sen}[k_y(n,m)y]$ $\cos[k_y(n,m)y]$	$\operatorname{sen}[k_z(n,m)z]$ $\cos[k_z(n,m)z]$

II. En coordenadas cilíndricas.

SOLUCIÓN TRIVIAL ($k(n)=0$): $\Phi(\rho,\phi,z)=(A+B\ln\rho)\,(C+D\phi)\,(E+Fz)$

SOLUCIÓN GENERAL ($k(n)>0$):

$$\Phi(\rho,\phi,z)=(E+Fz)\sum_n\left(A_n\,\rho^{k(n)}+B_n\,\rho^{-k(n)}\right)\left(C_n\cos[k(n)\phi]+D_n\operatorname{sen}[k(n)\phi]\right)$$

III. En coordenadas esféricas.

SOLUCIÓN TRIVIAL ($k(n)=0$): $\Phi(r,\theta,\phi)=(A+Br^{-1})\,(C+D\ln[\tan(\theta/2)])\,(E+F\phi)$

SOLUCIÓN GENERAL ($k(n)=n$):

$$\Phi(r,\theta,\phi)=\sum_n\left(A_n\,r^n+B_n\,r^{-(n+1)}\right)\sum_{m=0}^{n}P_n^m(\theta)\left(C_m\cos(m\phi)+D_m\operatorname{sen}(m\phi)\right)$$

donde $n=1,2,\ldots;\ m=0,1,\ldots,n$, y las funciones $P_n^m(\theta)$ de menor orden vienen dadas por la tabla:

$P_1^0(\theta)=\cos\theta$	$P_1^1(\theta)=\operatorname{sen}\theta$	
$P_2^0(\theta)=\cos 2\theta+1/3$	$P_2^1(\theta)=\operatorname{sen}2\theta$	$P_2^2(\theta)=\cos 2\theta-1$

SOLUCIONES TRIVIALES PARA MAGNETOSTÁTICA

$$\overline{H}(x,y,z)=\overline{1x}(A+Dy+Ez+Gyz)+\overline{1y}(B+Dx+Fz+Gxz)+\overline{1z}(C+Ex+Fy+Gxy)$$

$$\overline{H}(\rho,\phi,z)=\overline{1\rho}\,\frac{A+D\phi+Ez+G\phi z}{\rho}+\overline{1\phi}\,\frac{B+D\ln\rho+Fz+Gz\ln\rho}{\rho}+\overline{1z}(C+E\ln\rho+F\phi+G\phi\ln\rho)$$